# Adaptive Control

## Chapter 12: Indirect Adaptive Control

# Chapter 12
# Indirect Adaptive Control

**Abstract** Indirect adaptive control is a widely applicable adaptive control strategy. It combines in real-time plant model parameter estimation in closed loop with the redesign of the controller. Adaptive pole placement and its robustified version together with adaptive generalized predictive control constitute the core of the chapter. Adaptive linear quadratic control is also presented. Application of various strategies for the indirect adaptive control of a flexible transmission illustrates the methodology presented in this chapter.

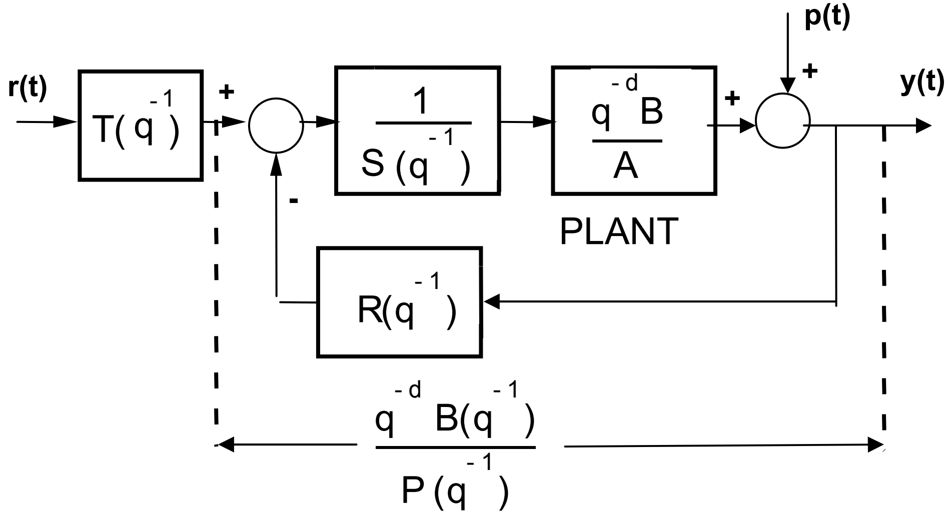
# Pole placement

The pole placement allows to design a R-S-T controller for

- stable or unstable systems
- without restriction upon the degrees of *A* and *B* polynomials
- without restrictions upon the plant model zeros (stable or unstable)
- but *A* and *B* polynomials should not have common factors (controllable/observable model for design)

*It is a method that does not simplify the plant model zeros*

# Structure




Plant: $$H(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A} \qquad B(q^{-1}) = b_1 q^{-1} + b_2 q^{-2} + \ldots + b_{n_B} q^{-n_B} = q^{-1} B^*(q^{-1})$$

# Pole placement

Closed loop T.F. (r $\rightarrow$ y) *(reference tracking)*

$$H_{BF}(q^{-1}) = \frac{q^{-d}T(q^{-1})B(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{q^{-d}T(q^{-1})B(q^{-1})}{P(q^{-1})}$$

$$P(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = 1 + p_1 q^{-1} + p_2 q^{-2} + \ldots.$$

Defines the (desired )closed loop poles

Closed loop T.F. (p $\rightarrow$ y) *(disturbance rejection)*

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})S(q^{-1})}{A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})} = \frac{A(q^{-1})S(q^{-1})}{P(q^{-1})}$$

Output sensitivity function

# Choice of desired closed loop poles (polynomial *P*)

$$P(q^{-1}) = P_D(q^{-1}) P_F(q^{-1})$$

**Dominant poles** **Auxiliary poles**

*Choice of $P_D(q^{-1})$(dominant poles)*

Specification in continuous time ($t_M$, M) $\longrightarrow$ $2^{nd}$ order ($\omega_0$, $\zeta$) $\xrightarrow[T_e]{\text{discretization}}$ $P_D(q^{-1})$

$$0.25 \le \omega_0 T_e \le 1.5$$
$$0.7 \le \zeta \le 1$$

*Auxiliary poles*

- *Auxiliary poles are introduced for robustness purposes*
- *They usually are selected to be faster than the dominant poles*

# Regulation( computation of *R(q⁻¹)* and *S(q⁻¹)*)

(Bezout) $$A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1}) = P(q^{-1}) \quad (*)$$

? ?

$$n_A = \deg A(q^{-1}) \quad n_B = \deg B(q^{-1})$$

*A* and *B* do not have common factors

*unique minimal solution for :*

$$n_P = \deg P(q^{-1}) \le n_A + n_B + d - 1$$

$$n_S = \deg S(q^{-1}) = n_B + d - 1 \qquad n_R = \deg R(q^{-1}) = n_A - 1$$

$$S(q^{-1}) = 1 + s_1 q^{-1} + \ldots s_{n_S} q^{-n_S} = 1 + q^{-1} S^*(q^{-1})$$

$$R(q^{-1}) = r_0 + r_1 q^{-1} + \ldots r_{n_R} q^{-n_R}$$

# Computation of *R(q-1)* and *S(q-1)*

Equation (*) is written as: $\boxed{Mx = p}$ $\longrightarrow x = M^{-1}p$

$$x^T = [1, s_1, \ldots, s_{n_S}, r_0, \ldots, r_{n_R}] \qquad p^T = [1, p_1, \ldots, p_i, \ldots, p_{n_P}, 0, \ldots, 0]$$

$$\begin{array}{c}
\overbrace{\hphantom{1 \quad 0 \quad \ldots \quad 0}}^{n_B + d} \qquad \overbrace{\hphantom{0 \quad \ldots \quad \ldots \quad 0}}^{n_A} \\
\underbrace{\left[\begin{array}{cccccccc}
1 & 0 & \ldots & 0 & 0 & \ldots & \ldots & 0 \\
a_1 & 1 & & . & b'_1 & & & \\
a_2 & & & 0 & b'_2 & & & b'_1 \\
 & & & 1 & . & & & b'_2 \\
 & & & a_1 & . & & & . \\
a_{n_A} & & & a_2 & b'_{n_B} & & & . \\
0 & & & . & 0 & . & . & . \\
0 & \ldots & 0 & a_{n_A} & 0 & 0 & 0 & b'_{n_B}
\end{array}\right]}_{n_A + n_B + d}
\end{array}
\left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\\1\\1\\1\end{array}}\right\} n_A + n_B + d$$

$b'_i = 0$ pour $i = 0, 1 \ldots d$ ; $b'_i = b_{i-d}$ pour $i > d$

Use of WinReg or *bezoutd.sci(.m)* for solving (*)

# Structure of $R(q^{-1})$ and $S(q^{-1})$

R and S may include pre-specified fixed parts (ex: integrator)

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1}) \qquad S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

$H_R$, $H_S$, - pre-specified polynomials

$$R'(q^{-1}) = r'_0 + r'_1 q^{-1} + \ldots r'_{n_{R'}} q^{-n_{R'}} \quad S'(q^{-1}) = 1 + s'_1 q^{-1} + \ldots s'_{n_{S'}} q^{-n_{S'}}$$

- The pre specified filters $H_R$ and $H_S$ will allow to impose certain properties of the closed loop.
- They can influence performance and/or robustness

$$A(q^{-1})H_S(q^{-1})\underbrace{S'(q^{-1})}_{?} + q^{-d}B(q^{-1})H_R(q^{-1})\underbrace{R'(q^{-1})}_{?} = P(q^{-1})$$

## Fixed parts ($H_R$, $H_S$). Examples

Zero steady state error ($S_{yp}$ should be null at certain frequencies)

$$S_{yp}(q^{-1}) = \frac{A(q^{-1})H_S(q^{-1})S'(q^{-1})}{P(q^{-1})}$$

*Step disturbance :* $H_S(q^{-1}) = 1 - q^{-1}$

*Sinusoidal disturbance :* $H_S = 1 + \alpha q^{-1} + q^{-2}$ ; $\alpha = -2\cos\omega T_s$

Signal blocking ($S_{up}$ should be null at certain frequencies)

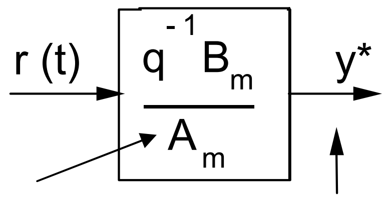
$$S_{up}(q^{-1}) = -\frac{A(q^{-1})H_R(q^{-1})R'(q^{-1})}{P(q^{-1})}$$

*Sinusoidal signal:* $H_R = 1 + \beta q^{-1} + q^{-2}$ ; $\beta = -2\cos\omega T_s$

*Blocking at* $0.5f_S$*:* $H_R = (1 + q^{-1})^n$ ; $n = 1,2$

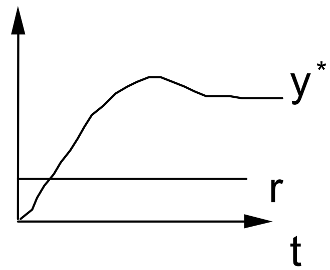
# Tracking (computation of $T(q^{-1})$ )

Ideal case




Tracking reference model ($H_m$)

desired trajectory for y (t)




$$H_m(q^{-1}) = \frac{B_m(q^{-1})}{A_m(q^{-1})}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots$$

$$A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \ldots$$

Specification in continuous time ($t_M$, M) $\longrightarrow$ $2^{nd}$ order ($\omega_0$, $\zeta$) $\xrightarrow[T_s]{\text{discretization}}$ $H_m(q^{-1})$

$$0.25 \le \omega_0 T_s \le 1.5$$

$$0.7 \le \zeta \le 1$$

*The ideal case can not be obtained (delay, plant zeros)*

*Objective : to approach y*(t)*

$$y^*(t) = \frac{q^{-(d+1)}B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

# Tracking (computation of $T(q^{-1})$ )

Build:

$$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})} r(t)$$

Choice of $T(q^{-1})$ :

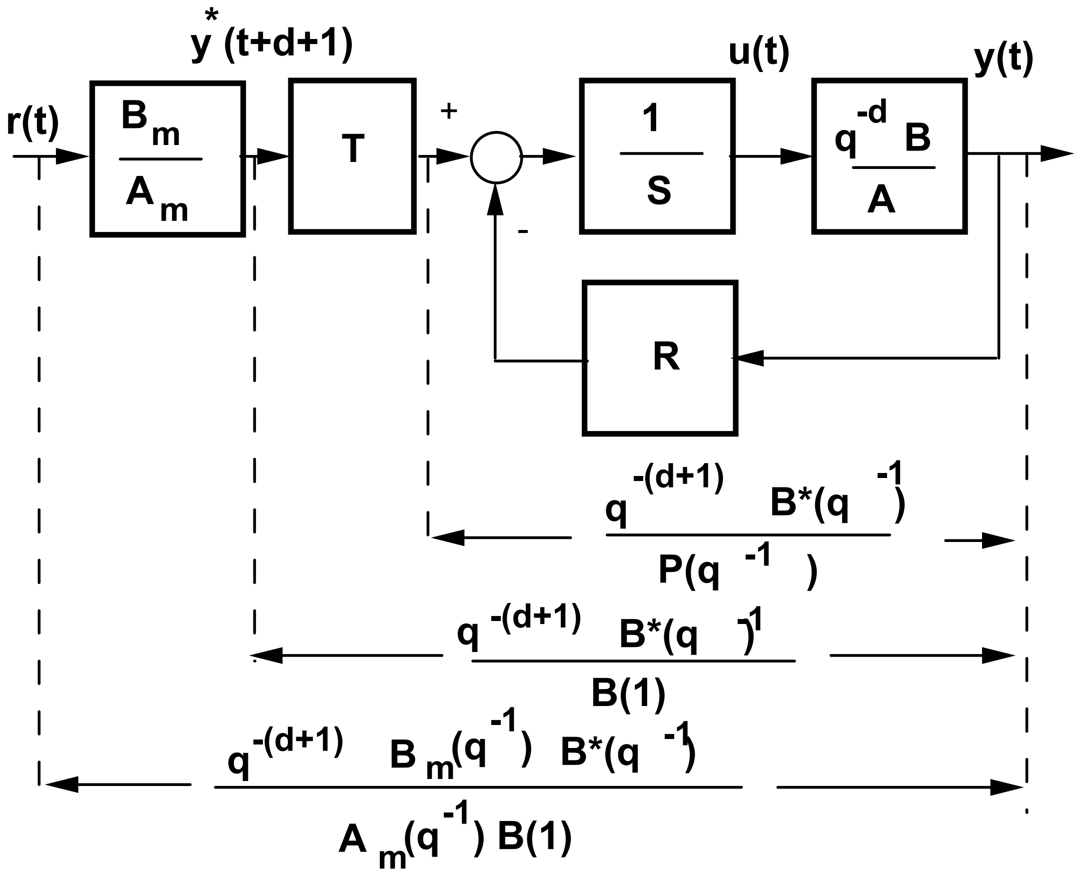
- Imposing unit static gain between $y^*$ and $y$
- Compensation of regulation dynamics $P(q^{-1})$

$$T(q^{-1}) = GP(q^{-1}) \qquad G = \begin{cases} 1/B(1) & if \quad B(1) \neq 0 \\ 1 & if \quad B(1) = 0 \end{cases}$$

F.T. $r \longrightarrow y$: $$H_{BF}(q^{-1}) = \frac{q^{-(d+1)} B_m(q^{-1})}{A_m(q^{-1})} \cdot \frac{B^*(q^{-1})}{B(1)}$$

*Particular case* : $P = A_m$ $$T(q^{-1}) = G = \begin{cases} \dfrac{P(1)}{B(1)} & if \quad B(1) \neq 0 \\ 1 & if \quad B(1) = 0 \end{cases}$$

# Pole placement. Tracking and regulation




$$S(q^{-1})u(t) + R(q^{-1})y(t) = T(q^{-1})y^*(t+d+1)$$

# Pole placement. Control law

$$u(t) = \frac{T(q^{-1})y^*(t+d+1) - R(q^{-1})y(t)}{S(q^{-1})}$$

$$S(q^{-1})u(t) + R(q^{-1})y(t) = GP(q^{-1})y^*(t+d+1) = T(q^{-1})y^*(t+d+1)$$

$$S(q^{-1}) = 1 + q^{-1}S^*(q^{-1})$$

$$\boxed{u(t) = P(q^{-1})Gy^*(t+d+1) - S^*(q^{-1})u(t-1) - R(q^{-1})y(t)}$$

$$y^*(t+d+1) = \frac{B_m(q^{-1})}{A_m(q^{-1})}r(t)$$

$$A_m(q^{-1}) = 1 + q^{-1}A_m^*(q^{-1})$$

$$\boxed{y^*(t+d+1) = -A_m^*(q^{-1})y(t+d) + B_m(q^{-1})r(t)}$$

$$B_m(q^{-1}) = b_{m0} + b_{m1}q^{-1} + \ldots \qquad A_m(q^{-1}) = 1 + a_{m1}q^{-1} + a_{m2}q^{-2} + \ldots$$

# Indirect adaptive control

At each sampling instant:

*Step I :* Estimation of the plant model $(\hat{A}, \hat{B})$
ARX identification (Recursive Least Squares)

*Step II*: Computation of the controller
Solving Bezout equation (for S' and R')

$$\hat{A} H_S S'(q^{-1}) + q^{-d} \hat{B} H_R R' = P$$

Compute:

$$R(q^{-1}) = R'(q^{-1}) H_R(q^{-1}) \qquad S(q^{-1}) = S'(q^{-1}) H_S(q^{-1})$$

$$T = \hat{G} P = \begin{cases} \hat{G} = \dfrac{1}{\hat{B}(1)} & if \quad \hat{B}(1) \neq 0 \\ \hat{G} = 1 \quad if \quad \hat{B}(1) = 0 \end{cases}$$

*Remark:*
*It is time consuming for large dimension of the plant model*

# Supervision

Estimation:

• Check if input is enough "persistently exciting"
(if not, do not take in account the estimations)

• Check if $\hat{A}$ and $\hat{B}$ are numerically "sound" (condition number)
(no close poles/zeros)

• If necessary, add external excitation (testing signal)

Control:

• Check if desired dominant closed loop poles are compatible with estimated plant poles

• Check robustness margins

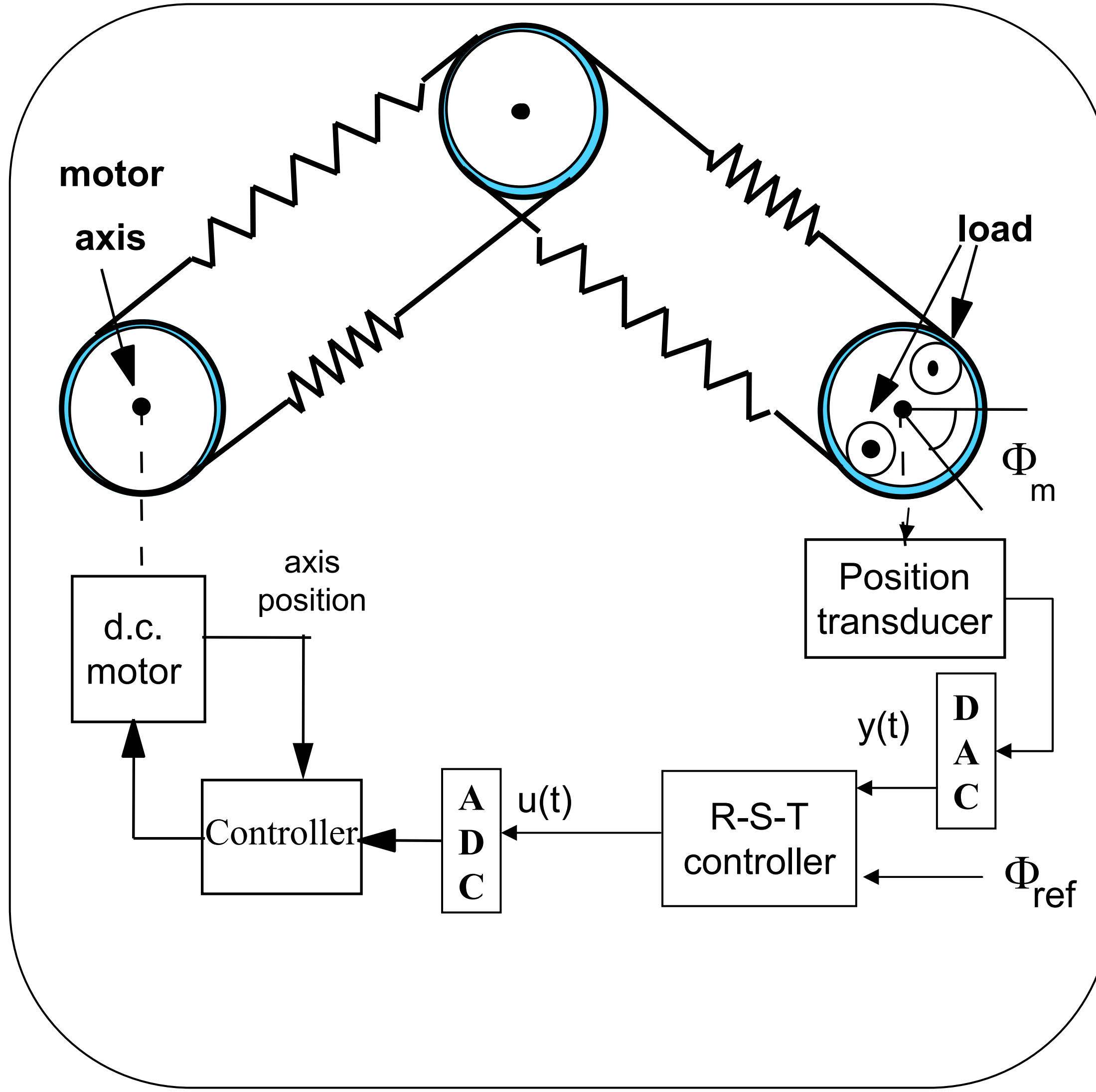
Additional problem:

• How to deal with neglected dynamics ?
(filtering of the data, robustification of PAA)

# Robust Control Design for Adaptive Control

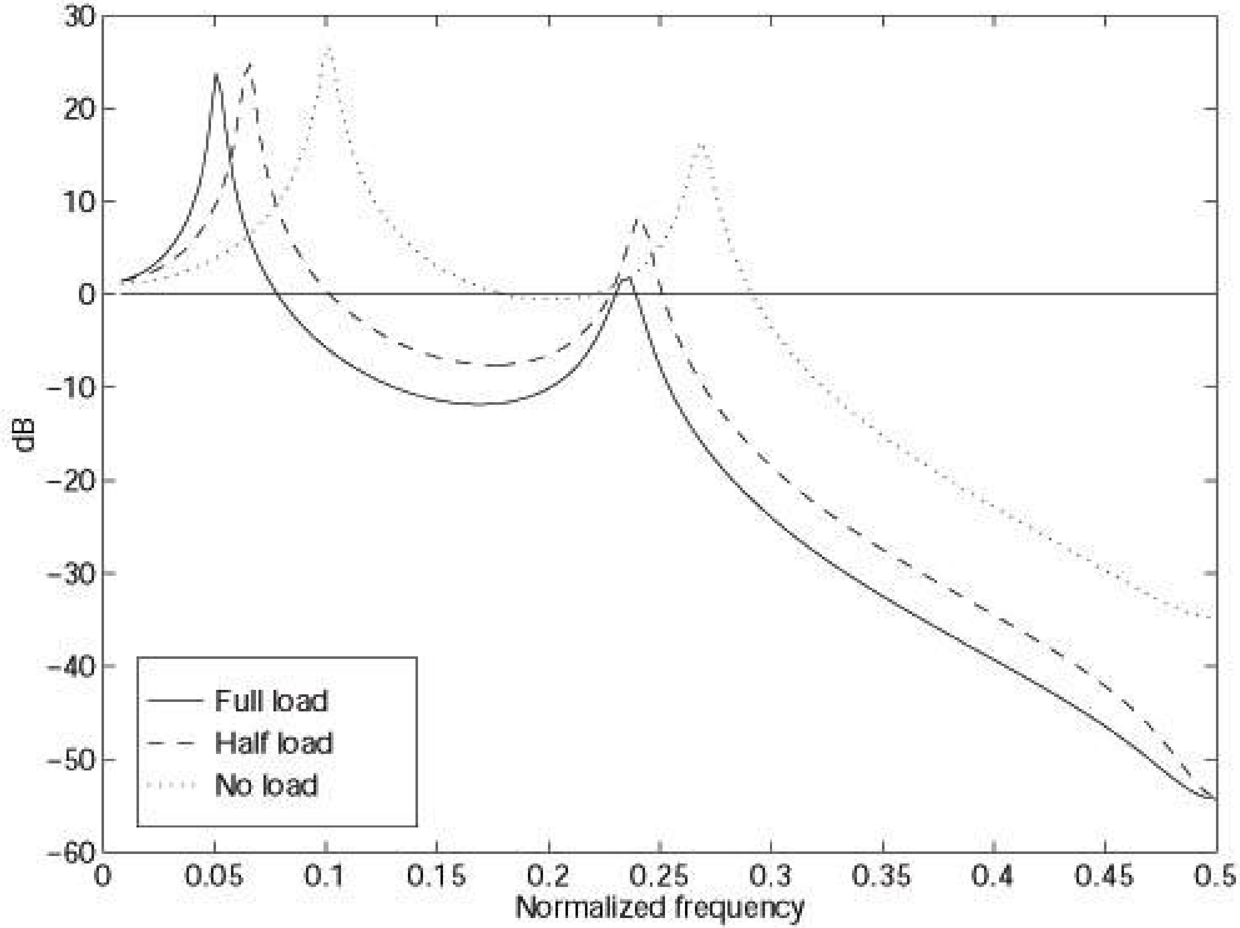
# Adaptive Control of a Flexible Transmission

The flexible transmission



# Adaptive Control of a Flexible Transmission

## Frequency characteristics for various load




*Rem.:* the main vibration mode varies by 100%

# Robust Control Design for Adaptive Control

**parameter variations** (low frequency) → **Adaptation**

**unstructured uncertainities** (high frequency) → **Robust Design**

**Basic rule** : The *input sensitivity function* ($S_{up}$) should be small in medium and high frequencies

*How to achieve this ?*

**Pole Placement :**

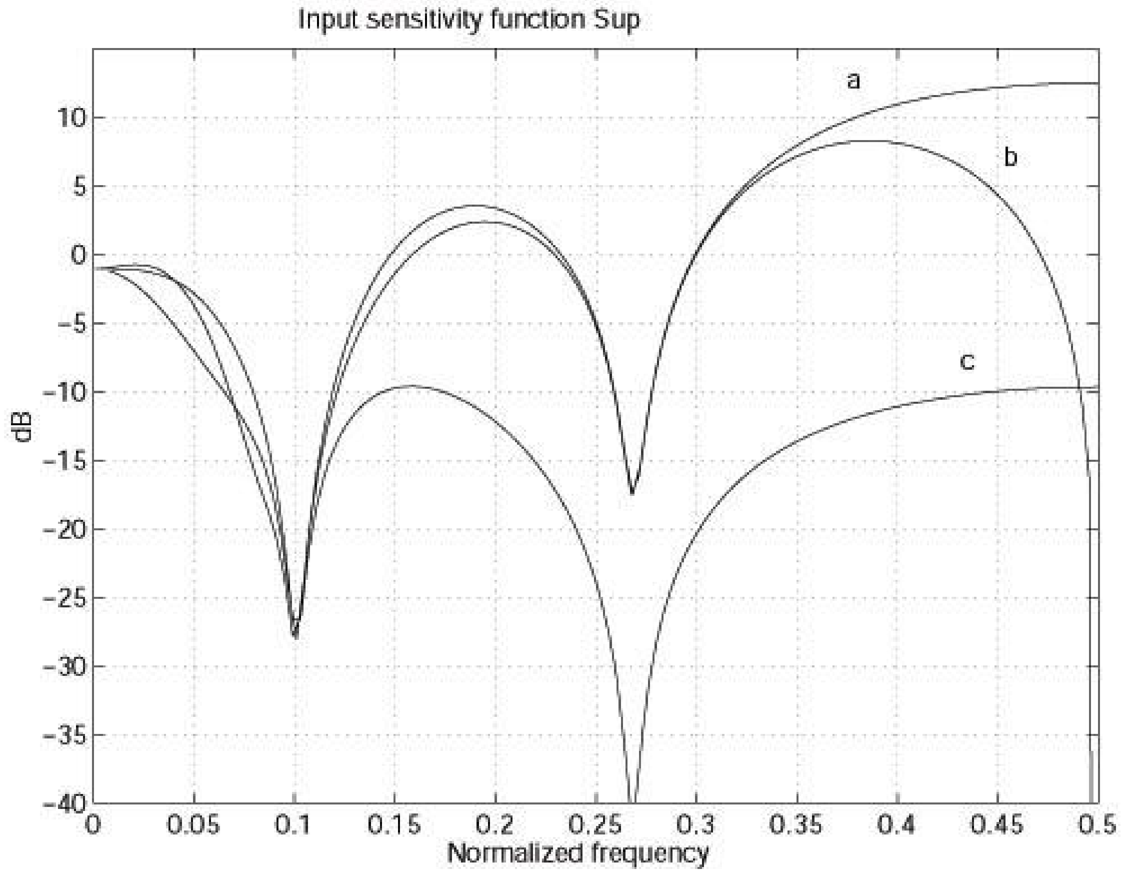
- Opening the loop in high frequencies (at $0.5f_s$)
- Placing auxiliary closed loop poles near the high frequency poles of the plant model

**Generalized Predictive Control :**

- Appropriate weighting filter on the control term in the criterion

# Robust Control Design for Adaptive Control

(Flexible Transmission)




a) Standard pole placement (1 pair dominant poles + h.f. aperiodic poles)

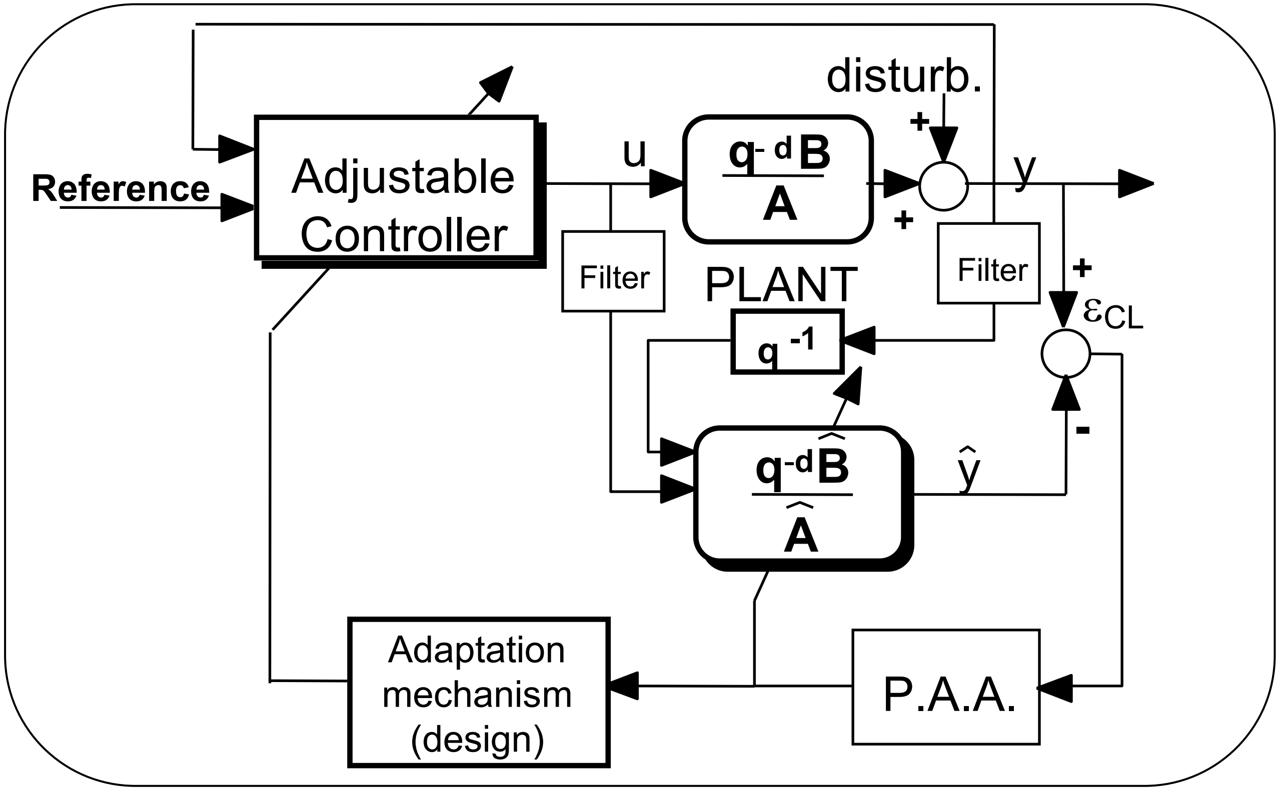
b) Opening the loop at $0.5f_S$ ($H_R = 1 + q^{-1}$)

c) Auxiliary closed loop poles near high frequency plant poles

## Parameter Estimators for Adaptive Control

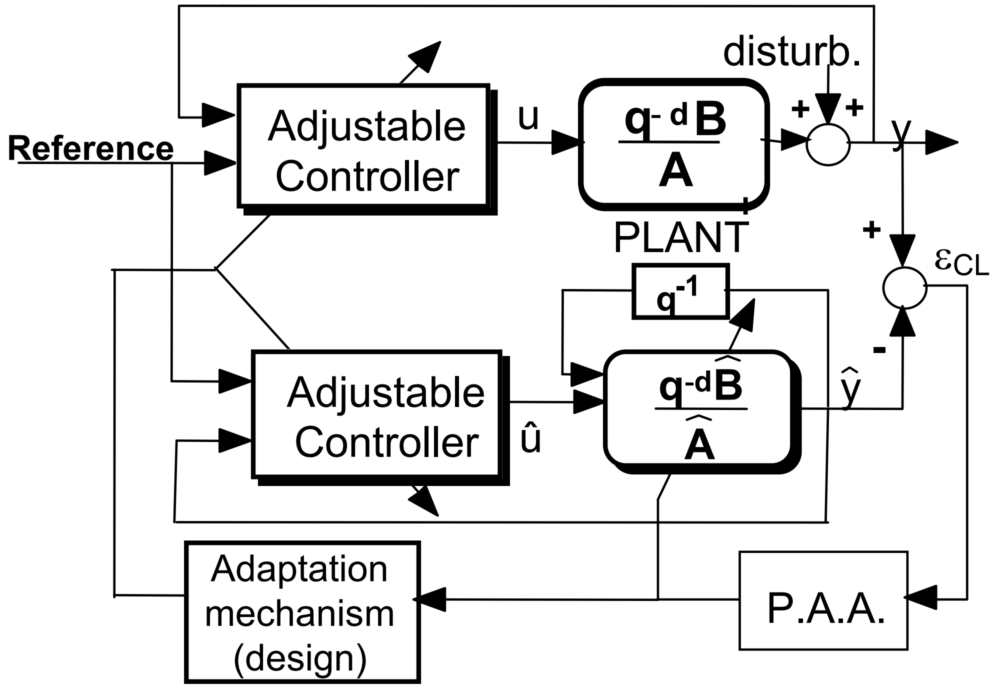
***Objective :*** *to reduce the effect of the disturbances upon the quality of the estimation*

# Classical Indirect Adaptive Control




- Uses R.L.S. type estimator (equation error)
- Sensitive to output disturbances
- Requires « adaptation freezing » in the absence of persistent excitation
- The threshold for « adaptation freezing » is problem dependent

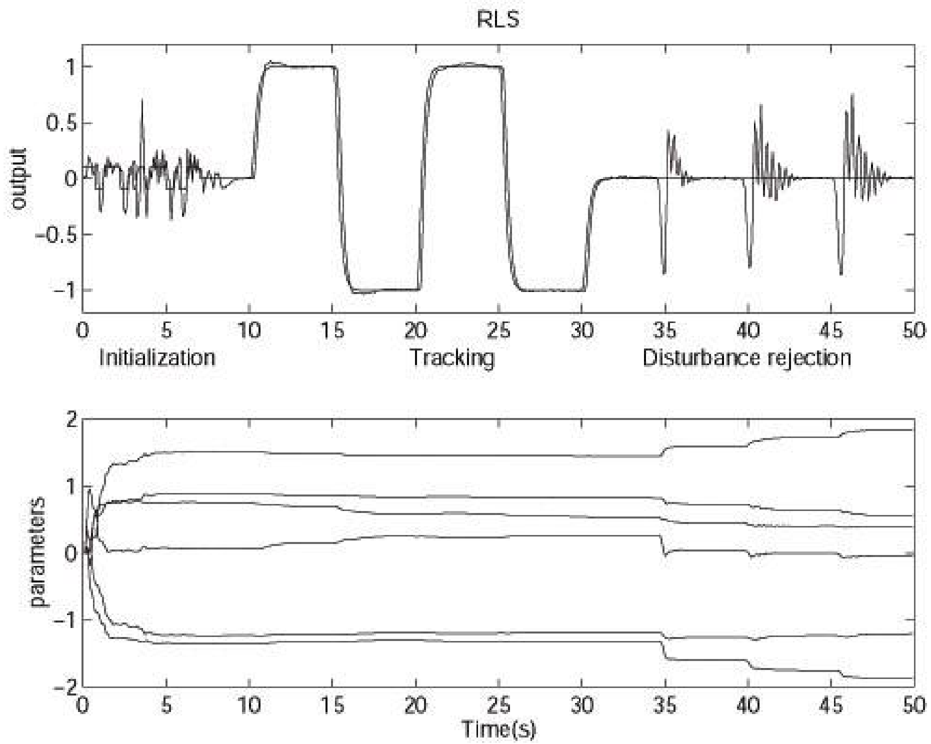
# Closed Loop Output Error Parameter Estimator for Adaptive Control



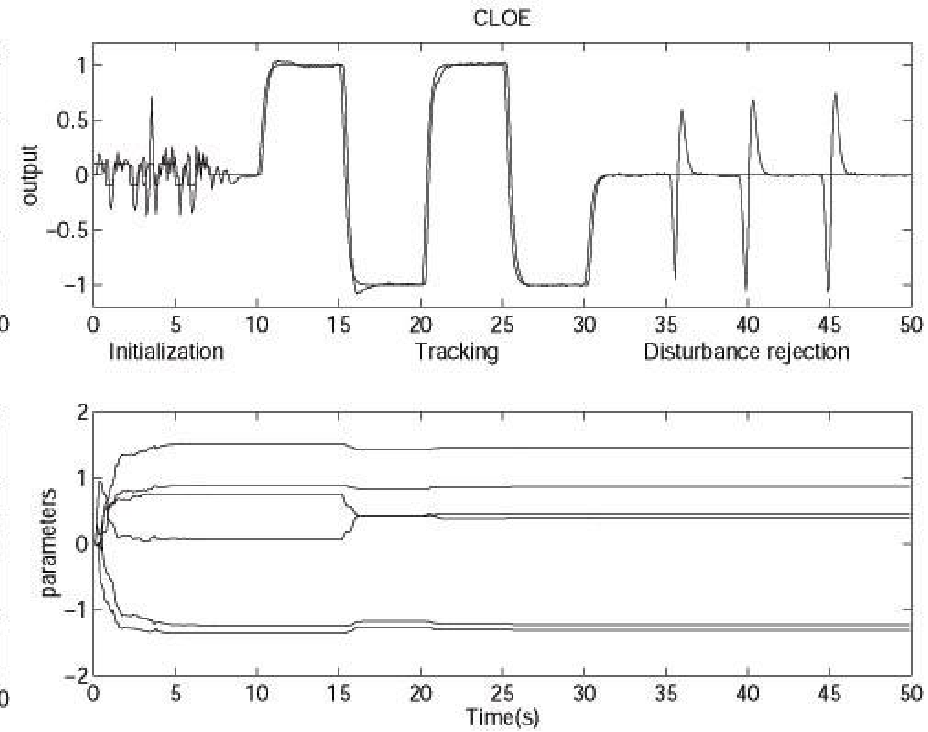

- Insensitive to output disturbances
- Remove the need for « adaptation freezing » in the absence of persistent excitation
- CLOE requires stability of the closed loop
- Well suited for « adaptive control with multiple models »

# Adaptive Control – Effect of Disturbances

Classical parameter estimator (filtered RLS)




CLOE parameter estimator




*Disturbances destabilize the adaptive system when using RLS parameter estimator (in the absence of a variable reference signal)*